Polar Volterra Series for Modeling of Power Amplifier and I/Q Modulator

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***Abstract*—Among the diverse parts and components that form a wireless data transmission system, the major player in energy consumption is the power amplifier (PA). Given that, many efforts are made to increase its efficiency. Among those, utilizing the PA at more efficient operation points. Nevertheless, these more efficient operation points tend to bring nonlinear behaviors to the signals being amplified. To try to mitigate that issue, many techniques are utilized, such as feed-back, feed-forward and, the one being addressed in this work, digital predistortion (DPD). In this study in specific, we handle a particular set of models, called the polar Volterra series (PVS). The work described here is the implementation and analysis of DPD techniques, mainly the PVS and modified variants, proposed by the authors, to handle the imperfections of I/Q modulation. The main contributions of this work are presenting a PVS model that is capable of handling I/Q imperfections and investigating the effects of different degrees of imperfection, for direct and inverse modelling.**

***Index Terms*—power amplifier, radiofrequency, I/Q modulators, digital predistortion**

I. INTRODUCTION

Wireless communication has become a staple of modern life. Cellphones, IoT and other wireless network based systems have all been essential to the development of the modern, interconnected society. Given their applications, the systems mentioned above have been pushed to ever more strict energetic restrictions, due to restraints in battery size and portability, making focus on reducing the energy consumption of those systems, without sacrifices in performance, be significant.

Amidst those attempts, one that constantly comes up, is the increase in the energetic efficiency of the power amplifier (PA), a key piece on data transmission systems [1]. PAs, in general, tend to consume significant amounts of energy, and therefore, increasing their efficiency means indirectly reducing the energy consumption of any give data transmission system.

However, there exists a trade off. Moving the operation point of a PA to its more efficient region usually comes with a nonlinear distortion of the amplified signal [2]. Considering that wireless data transmissions are very strict with regards to the linearity of transmitted signals, it usually means that the nonlinear operation point strategy is unfeasible. There have been many suggestions on how to tackle the issue, such as feed-back, feed-forward, predistortion and many others [3]–[9]. Among them, and the topic of this work, digital predistortion (DPD) is a very famous strategy [2]. This consists on modelling the distorting behaviour of the system and then treating the signal to an inverse of that behaviour. The natural distortion and the inverse predistortion cancel each other out and result into a undistorted version of the signal. This work seeks to study the modelling of such systems, specifically the ones that utilized I/Q modulation. I/Q modulation, in its ideal form simply segments the signal into two orthogonal signals, more fully occupying the band. However, due to the physical nature of modulators, I/Q modulators insert some noise into the signal [10]. This compromises the effectiveness of DPD techniques that compensates only PA distortions. In this work, we will implement and compare different models for predistortion, in regards to their efficiency to represent the distortions generated. Due to their large complexity, we will also tackle algorithmically pruned versions of those systems. The main contributions of the work are: presenting a Polar Volterra Series Model that is capable of handling a I/Q modulated signal; and investigating the effects, of direct and inverse modelling of those systems, of different levels of imperfections generated by an I/Q modulator.

II. PA BEHAVIOURAL MODELING

A. Polar Volterra Series (PVS) for PA

The Polar Volterra Series is a commonly utilized formative equation for the representation of power amplifiers [11]. It has excellent properties that allow it to model fading-memory, nonlinear systems, such as PAs. Its formative equation is defined by:

$$\tilde{y}(n) = \sum_{p_1=1}^{P1}\sum_{p_2=1}^{P2}\sum_{m_1=0}^{M}\cdots\sum_{m_{p1}=m_{p1}-1}^{M}\sum_{l_1=0}^{L}\cdots\sum_{l_{p2}=l_{p2-1}}^{L} \sum_{l_{p2+1}=0}^{L}\cdots\sum_{l_{2_{p2}-1}=l_{2_{p2}-2}}^{L} h_{p_1,p_2}(m_1,\ldots,m_{p_1},l_1,\ldots,l_{2p_2-1}) \prod_{k=1}^{p_1} a(n-m_k)\prod_{r=1}^{p_2} e^{j(\phi(n-l_r))}\prod_{s=p_2+1}^{2p_2-1} e^{-j(\phi(n-l_s))}, \quad (1)$$

where $P1$ and $P2$ are the powers by which the amplitude and phase information will be raised, respectively, and where M and L are the number of memory elements for amplitude and phase information, respectively. $a(n)$ and $\phi(n)$ are the amplitude and phase input for the sample time n, respectively, $\tilde{y}$ is the output and h indicates the Volterra kernel.

Sadly, this equation is unfit to represent systems that utilize non ideal I/Q modulation because I/Q imperfections make the model dependent not only on the complex input but also on the complex conjugate input. An alternative is suggested in Section III.

B. Reduction of coefficients using ascending algorithm

Due to the increasing complexity of PVS as $P1$, $P2$, M and L are increased, pruning algorithms usually are applied to reduce considerably the number of coefficients, without significantly reducing the quality of the model. In this work, the ascending algorithm was chosen to prune the system, due to its increase reliability [12]. The ascending algorithm starts by calculating a parameter of fidelity of each single coefficient, then, after selecting the one with best result, testing all combinations of two coefficients, the first being the one that has already been chosen, and the other one of the remainder coefficients. This process is repeated until the desired number of coefficients is achieved.

The metric of fidelity, in this case, is the normalized mean square error, which is given by:

$$NMSE = 10\log_{10}\left\{\frac{\sum_{n-1}^{N}|y_n^{des}-y_n^{cal}|^2}{\sum_{n-1}^{N}|y_n^{des}|^2}\right\}, \qquad (2)$$

where y_n^{des} is the measured output at the time sample n and y_n^{cal} is the estimated output at the time sample n calculated from any of the given bases, and N is the number of samples in the dataset. It is important to note that the NMSE, as a parameter of difference, that is between the measured and calculated data, and being of logarithmic nature, is considered better as the number becomes more negative.

III. PROPOSED POLAR VOLTERRA SERIES M (PVSM)

The modeling of amplitude and phase mismatches in I/Q modulation demands for contributions in which the number of negative input phases exceeds the number of positive input phases for one. This is the formative equation for the PVSM:

$$\tilde{y}(n) = \sum_{p_1=1}^{P1}\sum_{p_2=1}^{P2}\sum_{m_1=0}^{M}\cdots\sum_{m_{p1}=m_{p1}-1}^{M}\sum_{l_1=0}^{L}\cdots\sum_{l_{p2}=l_{p2-1}}^{L}$$
$$\sum_{l_{p2+1}=0}^{L}\cdots\sum_{l_{2_{p2}-1}=l_{2_{p2}-2}}^{L} h_{p_1,p_2}(m_1,\ldots,m_{p_1},l_1,\ldots,l_{2p_2-1})$$
$$\prod_{k=1}^{p_1}a(n-m_k)\prod_{r=1}^{p_2}e^{-j(\phi(n-l_r))}\prod_{s=p_2+1}^{2p_2-1}e^{j(\phi(n-l_s))}, \qquad (3)$$

where $P1$, $P2$, M and L and h represent the same thing as in (1). Using (1) and (3) in conjunction will allow to represent both PA and I/Q distortions.

In the presence of imperfections of the modulator, the output of the modulator depends on the non-conjugate output, x, and the conjugate output, x^*. Combining this with nonlinearities of the PA, (1) takes into account the dependency of the non-conjugate input (an additional positive phase), meanwhile (3) takes into account the dependency of the conjugate input (an additional negative phase).

IV. SIMULATION RESULTS

The measurements were taken from RF transmission simulations, using single band for the PA model, with and without the presence of memory, that will be hereafter called measured data. A 5th order polynomial, whose parameters are shown in [13], is adopted to mimic the nonlinear PA behavior. The memory effects are added by placing a FIR filter before the polynomial. The FIR is of third order, with coefficients 0.8372, 0.1022 and 0.0578, The signal used was a standard LTE OFDMA, discrete in time, with bandwidth of 10 MHz, with sampling frequency of 61.44 MHz.

After that, the sampled signals were treated by an I/Q modulation equation given by:

$$s(n) = \mathbb{R}\{y(n)[\cos(2\pi f_c t) - (1+\alpha)j\sin(2\pi f_c t+\varphi)]\}, \qquad (4)$$

where f_c is 1.2288 GHz, φ is the difference introduced between the cosine and the sine, and α is the difference in amplitude. The different values of α and φ are reported in Table I.

TABLE I
DESCRIPTION OF DATASETS

	Memory	α	φ
Dataset 1 (D1)	Yes	0.20	$\pi/3$
Dataset 2 (D2)	Yes	0.04	$\pi/15$
Dataset 3 (D3)	Yes	0.02	$\pi/30$
Dataset 4 (D4)	No	0.20	$\pi/3$
Dataset 5 (D5)	No	0.04	$\pi/15$
Dataset 6 (D6)	No	0.02	$\pi/30$

The values of α and ϕ were chosen arbitrarily, with the motivation of presenting a case with many imperfections and other cases with few imperfections. Considering that even when both α and ϕ are small simultaneously PVSM still contributed significantly, it is implicit that, in a case where only α or ϕ are big, PVSM will also contribute, therefore no simulations with high α but small ϕ or big ϕ and small α were made.

Transient simulations with a sampling frequency of 12.288 GHz and 409,600 samples are done. Ideal demodulation followed by downsampling is performed to provide the baseband envelopes at a sampling frequency of 61.44 MHz. After splitting each dataset in extraction and validation, the extraction datasets were utilized to retrieve kernels for validation. For the $D1$ to $D3$, $M = 1$ and $L = 1$ were utilized. Meanwhile, for $D4$ to $D6$, due to their lack of memory, $M = 0$ and $L = 0$ were utilized. Following this, an ascending algorithm was utilized to prune the coefficients to ten, since after around eight or so coefficients, the contributions to the NMSE diminished considerably, as can be observed in Figure 1.

This figure allows us to evaluate to which degree increasing the number of coefficients, given PVS+PVSM, increases the performance, in terms of NMSE, of the models. Thus, verifying the benefits of utilizing PVSM only needs to be tested

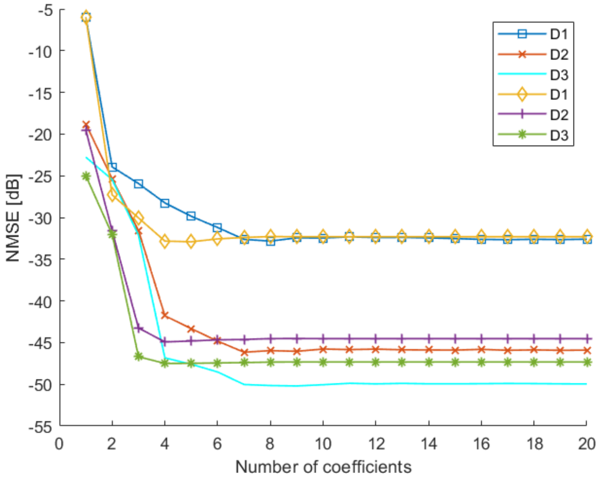

Fig. 1. Validation NMSE for increasing number of coefficients, direct modelling of PVS+PVSM.

to that point, that is, of eight of so coefficients. A margin of two coefficients is added for the sake of visualization. To verify the benefits of utilizing PVSM in conjunction with PVS, the number of PVSM coefficients were restricted to zero, and then gradually increased. First zero coefficients, then subsequently one until five out of the ten coefficients. Allowing for larger amounts (e.g. six) did not render further increase of the number of coefficients, therefore further tests were not made. These restrictions enable the measurement of increasing the participation of PVSM to the modelling of the system. After such restrictions were applied, an ascending algorithm was applied from one to ten coefficients and the resulting NMSE for each number, for each of the datasets was plotted. The results are shown in Figures 2 and 3.

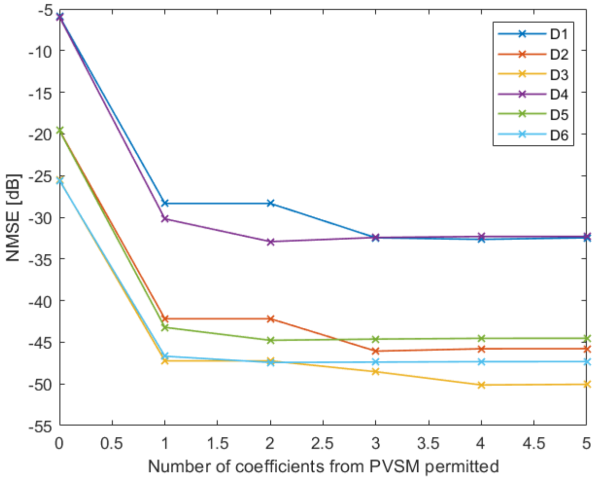

Fig. 2. Validation NMSE (at ten coefficients) for different limits of PVSM coefficients, direct modelling.

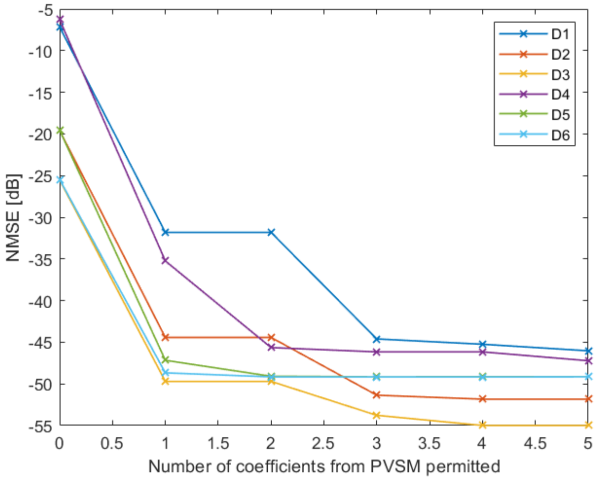

Fig. 3. Validation NMSE (at ten coefficients) for different limits of PVSM coefficients, for inverse modelling.

Figures 2 and 3, containing performance on the y-axis and information related to the restriction of PVSM coefficients on the x-axis, allow us to evaluate the impact of increasing the participation of PVSM to the improvement of the model. Moving along the x-axis, which is equivalent to increasing the number of PVSM coefficients on a ten coefficients model, it can be observed that indeed, the NMSE decreases, and as it has been stated before, this is equal to an increase of performance.

For all six datasets, the quality of the model was reduced if the coefficients of PVSM was limited. In Tables II and III we can see the worse and the best scenario for direct and inverse modelling.

TABLE II
IMPROVEMENTS OF PVS+PVSM FOR DIRECT MODELLING

Improvement	Dataset	NMSE gain
Largest	D1	26.6 dB
Smallest	D6	21.7 dB

TABLE III
IMPROVEMENTS OF PVS+PVSM FOR INVERSE MODELLING

Improvement	Dataset	NMSE gain
Largest	D4	40.96 dB
Smallest	D3	29.5 dB

Another way to visualize the improvements obtained is by plotting what is known as an AM-AM graph, which shows the relationship between the input amplitude and the output amplitude. Such plots are shown in Figures 4 and 5. It can be noticed that, for both direct and inverse modelling, the inclusion of five coefficients from PVSM substantially improved the quality of the estimations. This can be observed by comparing the AM-AM characteristic with and without PVSM and the measured data. The measured data and the 'with PVSM' data approach significantly, while the 'without

PVSM‘ data is considerably different, which reflects in an inability of the basis to model the behavior of the PA, which means that it is not a good model for DPD techniques.

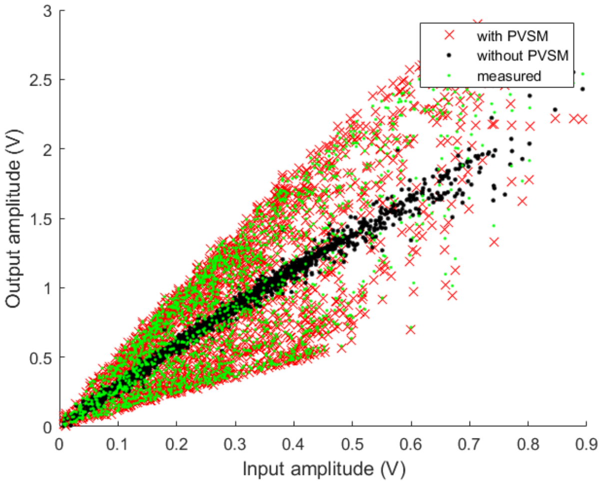

Fig. 4. AM-AM characteristics for direct modelling of D1.

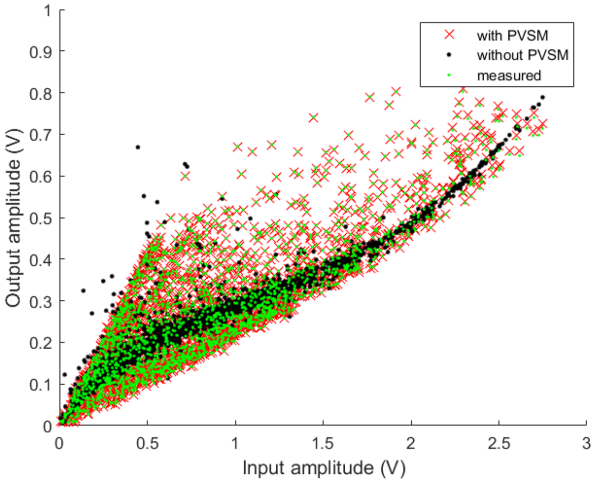

Fig. 5. AM-AM characteristics for inverse modelling of D1.

V. CONCLUSION

It can be observed, in all cases, that is, for direct and inverse modelling, with or without memory and for any amount of I/Q imperfections, that there were significant improvements as we increase the number of coefficients from PVSM. In other words, the suggested model, PVSM, indeed helps in the modelling of I/Q modulated PA signals. Other than that, it is interesting to remark, that for even as we decreased the I/Q imperfections, PVSM still provided significant improvements.

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